VORTEX DRIFT INDUCED BY AN ELECTRIC FIELD IN EXCITABLE MEDIA

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Abstract

Our simulations with the two variable Oregonator model have shown that the width of the tip of rotating vortex is increased (decreased) when it propagates to the positive (negative) electrode. This results in increasing (decreasing) the tangential growth velocity and in arizing of a perpendicular component of the drift defined by the chirality of rotating spiral. We propose a new kinematical model where the tangential growth velocity depends on the wave width rather than on the curvature of the wavefront.

Investigations of electric field influence on chemical active media have shown (*Schmidt et al.* 1977), that a wave propagating to the positive electrode is accelerated while a wave propagating to the negative one is decelerated. It is quite clear that this effect would give rise to a drift of rotating vortex toward the positive electrode. The effect of electric field on spiral waves and this drift have really been found experimentally (*Pérez- Muñuzuri et al.* 1991). Unexpectedly, a component of the drift, perpendicular to electric field direction, was found too. This component was comparable, or even greater, than the parallel component of the drift.

We propose a mechanism (*Muñuzuri et al.* 1993) which can induce a perpendicular component of the drift. It is based on the following idea. The width w of a wave moving with greater velocity should be increased, since $w = \tau V$, where V is the wave velocity and τ is the characteristic time scale of chemical kinetics. For a rotating vortex, it means that the width of its tip should be periodically modulated, increasing when the tip moves to the positive electrode and decreasing when it moves to the negative one. This in turn should affect the wave propagation. In particular, the increase of the width of the vortex tip should result in increasing the tangential growth velocity of the wave break and in arising a perpendicular component of the drift defined by the chirality of rotating spiral.

To investigate the validity of this hypothesis, we first check that the tip width is periodically modulated, and its increase and decrease occur in proper orientation with respect to the electric field. Then, we compare the tangential growth velocities of oppositely oriented wave breaks in electric field. And at last we reconstruct the drift pattern with a kinematical model with a tangential growth velocity defined by the width of the vortex tip.

We studied numerically the two variable Oregonator model (*Keener et al.* 1986):

$$\partial u/\partial t = F/\varepsilon + D_u \Delta u + E_x \nabla_x u \qquad (1)$$
$$\partial v/\partial t = \Phi + D_v \Delta v$$

Here variables u and v represent the dimensionless concentration of $HBrO_2$ and of ferriin, respectively. D_u and D_v are the dimensionless diffusion coefficients of u and v. E is the electric field. Functions, $F = u - u^2 - fv(u-q)/(u+q)$ and $\Phi = u - v$; q, f and ε are parameters depending on the chemical kinetics. The particular values of these parameters used in our approach were $q= 0.002$, $f= 1.4$ and $\varepsilon= 0.05$

In order to achieve faster computation times, we supposed a Laplacian involving five grid points, the closest neighbours. The obtained waves were indistinguishable from those generated by using a most precise discretization.

Fig.1 demonstrates electric field influence on a rotating vortex. It is seen that electric field increases the core size of the vortex and initiates vortex drift both in the parallel and in the perpendicular direction. In Fig.2, enlarged images of the tip are shown in two different positions separated for half a period of rotation. The width of the tip is seen to be increased (decreased) when it moves to the positive (negative) electrode.

Changes of the wave width should influence wave velocity because they affect the diffusion flux into an area element ahead of the front. Standard kinematical model (*Zykov* 1987) ignores this effect. The improvement of the model given by *Meron et al.* (1988). Their model describes the influence of the width on normal velocity only. We studied normal V_N and tangential growth V_g velocities of a broken wave front by comparing two positions of the wave break as shown in Fig.3. While there is no problem to measure the normal velocity, the tangential growth velocity was measured only during first 250 time steps; at longer time the curling is pronounced and the obtained velocity will present normal and tangential components.

In Fig.3 the electric field was applied along x-axis, so it directly affects normal velocity only and not the tangential growth velocity. We observed nevertheless (Table I) that not only the normal velocity but the tangential growth velocity was changed as well as the wave width.

These results can be easily transposed to the rotating vortex. The vortex tip is affected depending on its relative position in the electric field. The thickness of the tip is enlarged when the tip propagates in the direction of the electric field and decreases when it propagates in the reverse direction as it was shown in Fig.2. Then the tangential growth velocity should behave in the same way. This results in the vortex drift with a perpendicular component defined by the chirality of the rotating spiral.

Note that the characteristic time scale of chemical kinetics τ as estimated from Table I ($\tau \cong 0.26 \pm 0.02$ tu) is not influenced by the intensity and direction of the electric field. Its independence on electric field permits us to write a simple kinematical model of the vortex drift in an electric field.

The normal velocity V_N can be represented as $V_N = V_0 + \beta \sin \vartheta$, where ϑ is the angle between the tip and E and the tip width of rotating vortex is $w = w_0 + \alpha \sin \vartheta$, where α and β are parameters depending mainly on E. V_0 and w_0 are the normal velocity and width of the front without electric field.

We model the tangential growth velocity as proportional to w,

$$V_g = \gamma\ w \qquad (2)$$

For this model, the position (x,y) of the tip is described as

$$\begin{aligned} \dot{x} &= -\ V_g \sin \vartheta - V_N \cos \vartheta \qquad (3) \\ \dot{y} &= -\ V_g \cos \vartheta + V_N \sin \vartheta \end{aligned}$$

where $\vartheta = \omega t$ (ω is the frequency of rotation). From these equations, the drift velocity V_{dr} and its angle φ can be easily obtained as

$$V_{dr} = \frac{1}{2} \sqrt{\beta^2 + \gamma^2\ \alpha^2}, \qquad \tan(\varphi) = \gamma\alpha/\beta \qquad (4)$$

Fig.4 shows the x and y position of the tip as functions of time, obtained from the numerical integration of Eq.(3) compared with the Oregonator's calculations. It is seen that the proposed hypothesis is not only physically relevant but also gives similar dynamical patterns.

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Table I

Influence of the electric field on the wave break characteristics.

	V_N	V_g	w
E = 0	3.3	1.8	13.5
E = -1	2.5	1.7	10.5
E = +1	4.4	1.9	16.5

V_N - normal velocity, V_g - tangential growth velocity, w - thickness of the wave. Same parameters as in Fig.1.

FIGURE CAPTIONS

Figure 1: Drift of the vortex in an electric field parallel to the x-axis. Black - the vortex with E=0. Gray - the vortex 67 time units after switching on the electric field (E=+1, positive electrode at the left). Two variable Oregonator model [5], ε = 0.1, f = 1.4, q = 0.002, D_u = 1, D_v = 0.6, $\Delta t = 10^{-3}$ and Δx = 0.2.

Figure 2: Vortex tip **(a)** no electric field, E = 0; **(b)** E = 1; **(c)** E = -1. All the images are calculated from the same initial conditions; two positions of the tip separated for half period of rotation are shown (black and grey). Note that the tip width is increased when it is moving to the positive electrode (parameters are the same as in Fig.1).

Figure 3: Normal V_N and tangential growth V_g velocities. Solid line - initial position of a broken wave front (t=0), dashed line - position at t=0.25.

The broken wave was created as follows. The wave was moving to the right in a medium 200x35 su. Then at t = 0, Y -size of the medium was increased to 50 su (same parameters as in Fig.1, but D_V=0).

Figure 4: x (a) and y (b) position of the tip of the spiral wave drifting in the electric field.

Solid line - Oregonator model, dashed line - kinematical model.

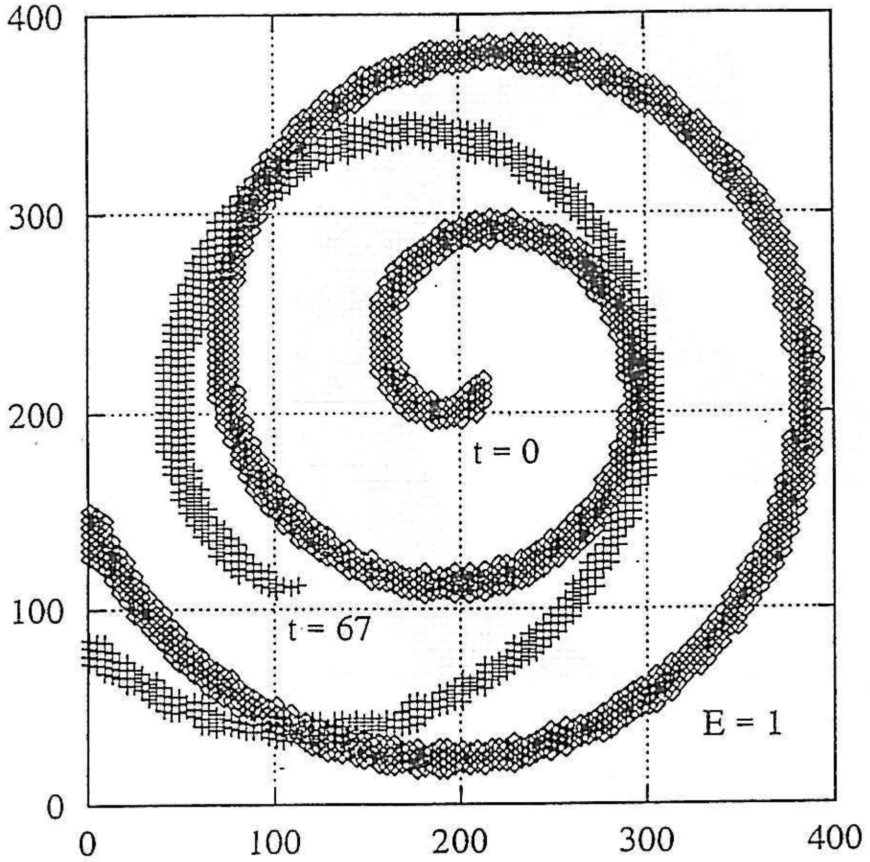


Fig. 1

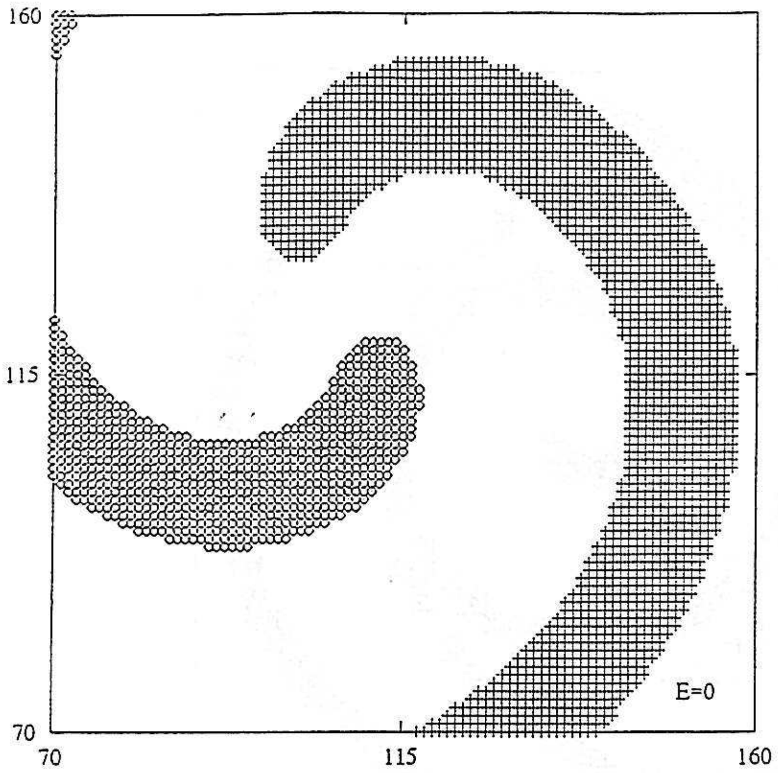


Fig 2a

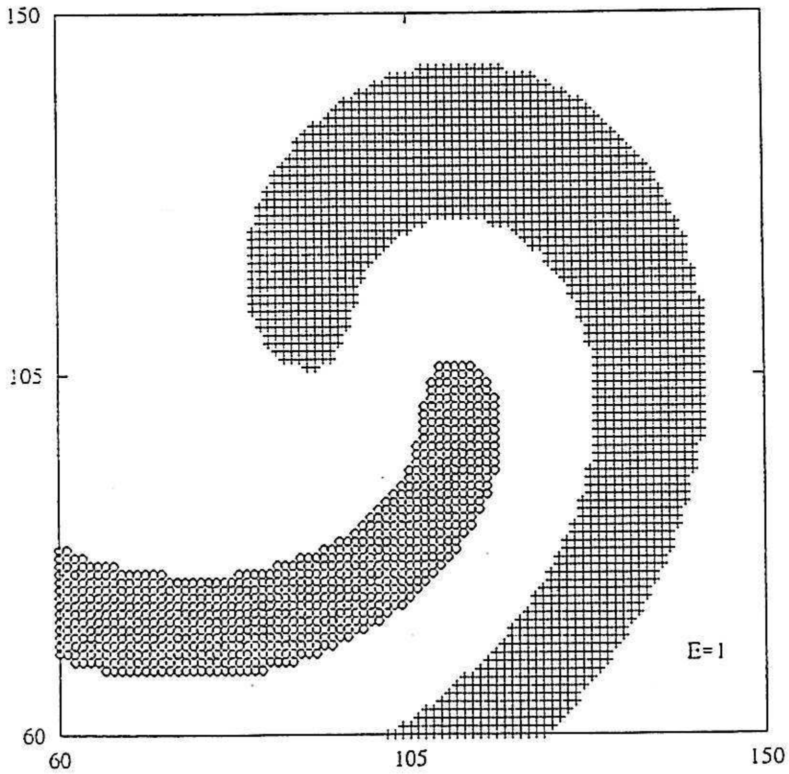


Fig 2b

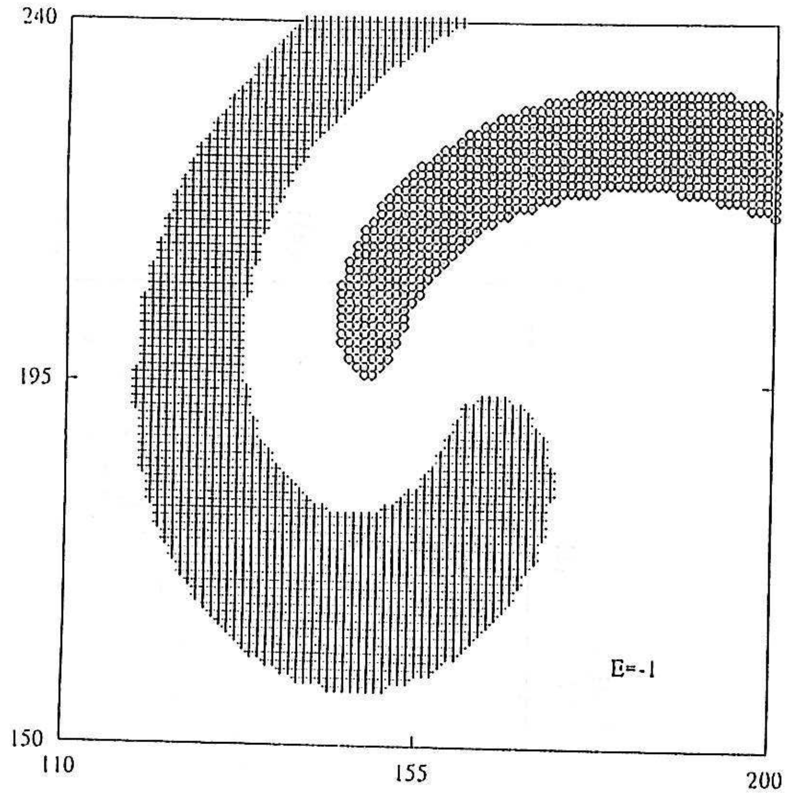


Fig 2c

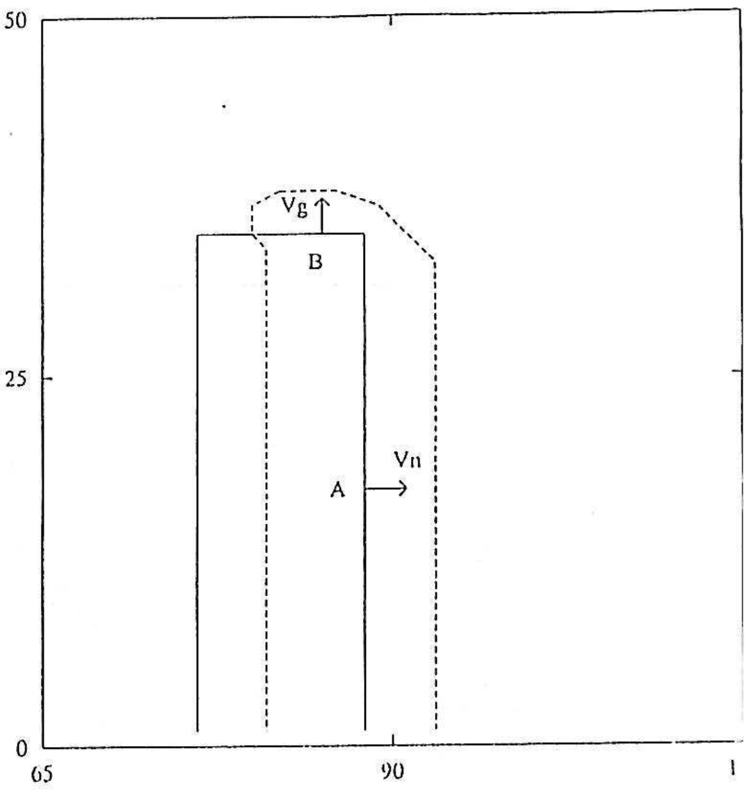
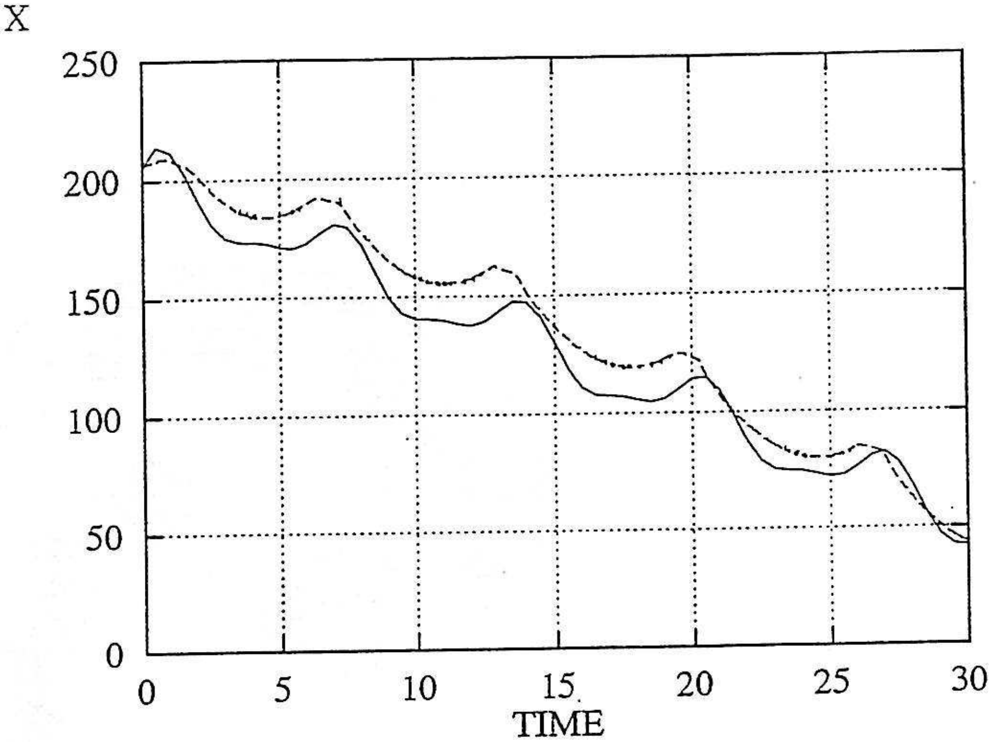


Fig 3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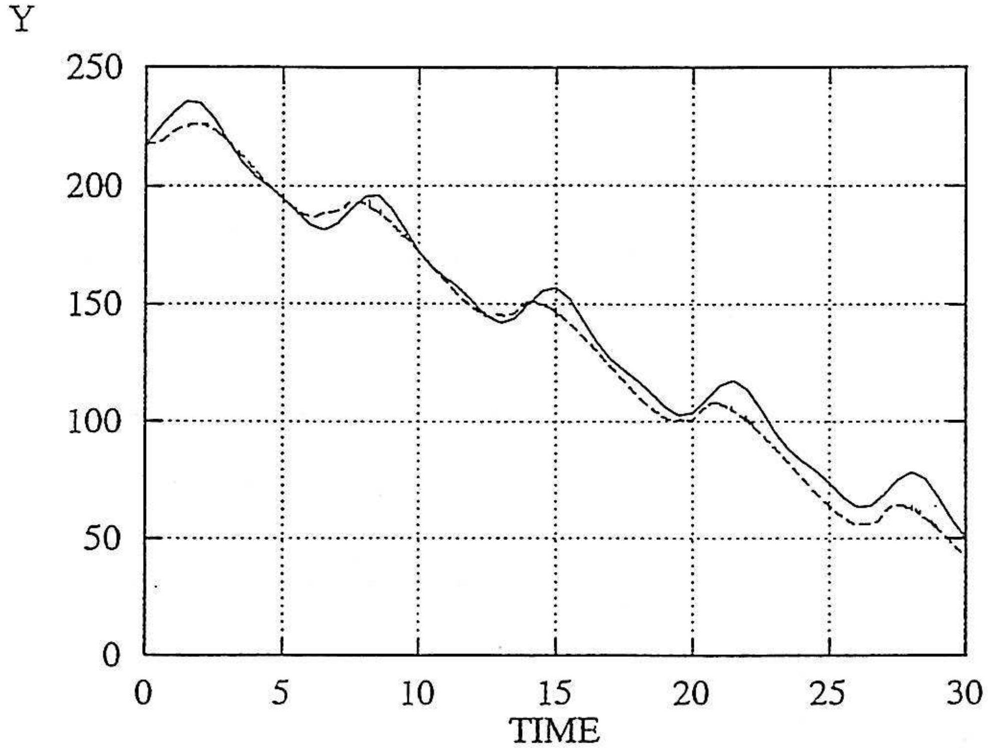
TIME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