Non-linear forecasting of cats eye movement time series

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ABSTRACT

The method of non-linear forecasting of time series was applied to the spontaneous eye movements of the cat in order to determine whether that complex sensori-motor system exhibits stochastic or chaotic behavior. Two particular states, normal awake cat and pathologic nystagmus were analysed. This behavior appeared to be chaotic in contrast with the normal case.

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A. Introduction

The study of eye movements presents a unique opportunity in understanding how the brain controls behavior. The functions of the eye movements are of two principal types. The first one is to keep images steady on the retina during the movements of the head (this is the vestibulo-ocular-reflex) and during the movement of the visual surround field (this is the optokinetic reflex). The second one is to change the line of sight by making saccades. The spontaneous eye movements are under voluntary and involuntary controls that consist of several levels of a parallel neural network organisation. The performance of these movements has been found to be impaired in many different neurological and psychiatric diseases.[1] If the oculomotor processes are relatively well understood on the biomechanical and neurophysiological points of view, the global dynamical behavior of the eye movements remains unexplored. Therefore it should be interesting to find out whether the eye movement behavior follows a deterministic chaotic dynamics or a random phenomenon. As a first approach to this question,we present here the application of non-linear forecasting to the horizontal cat eye movement. This procedure is performed on spontaneous ocular movements during two different cases : for a normal awake cat and in pathological eye nystagmic state produced by a microinjection of a chemical subtance (NMDA receptors antagonist) into the oculomotor neural integrator located in the brainstem.

For a few years the formalism of chaotic attractors has been applied to tentatively quantify EEG[2] and ECG[3] patterns. The method initialy used was the Grassberger-Procaccia algorithm[4]. However this method, at first sight easily applicable to experimental time series, encounters criticism as it allows analyst's subjectivity (choice of the time lag, determination of the scaling region) and as accuracy and computation time have contradictory requirements on the number of experimental data N. Another way of testing time series is non-linear forecasting[5],[6]. The underlying idea of this method is that distinction between dynamical chaos and statistical noise can be made by the comparison of predicted and actual trajectories. The future of a deterministic chaotic time series can be forecasted in a short time interval but the accuracy of the forecast falls off with increasing time. For uncorrelated noise on the other hand the forecasting accuracy is roughly time independant.

B. Method

The non-linear forecasting was performed as follows[5],[6]. From the original time series x_t an embedding space of dimension n was constructed whose points $\underline{X}_t$ are

$$\underline{X}_t = (x_t, x_{t-\tau}, x_{t-2\tau}, ..., x_{t-(n-1)\tau}) \tag{1}$$

where τ is a time delay, $t \in [1, 2N]$

The data set is divided in two parts. The first N data points $\underline{X}_t$ serve as a pattern by keeping track of where they move p time steps in the future to make the predictions $\underline{Y}_{j+p}$ of the evolution p time steps in the future for the next N points.

We adopt [7] for the prediction $\underline{Y}_{j+p}$ for p time steps in the future for a given point $\underline{X}_j$, $j > N$, the following expression

$$\underline{Y}_{j+p} = \sum_{i=1}^{n_d+1} \underline{\tilde{X}}_{k_i+p} \overline{e}^{\alpha|\underline{\tilde{X}}_{k_i} - \underline{X}_j|} \tag{2}$$

where $k_{i+p} < N$, $|...|$ is the Euclidean distance and $\tilde{X}_{k_i}$ is one of $n_d + 1$ closest neighbors (in the first half of the data $k_i < N$) of X_j, α is a constant.

The accuracy of the prediction is evaluated by the correlation coefficient between forecast y_{j+p} (first component of $\underline{Y}_{j+p}$) and actual time series x_{j+p}

$$C(p) = \frac{< y_{j+p}x_{j+p} > - < y_{j+p} >< x_{j+p} >}{\sigma_y \sigma_x}$$

where the average $<>$ is performed over N points, $N + 1 < j + p < 2N, \sigma_x$ and σ_y are the corresponding standard deviations.

For example, to compute the correlation coefficient for predictions one time step in the future ($p = 1$) points $\underline{X}_1$ to $\underline{X}_N$ are used to predict the values obtained by applying the iterative map (2) to points X_{N+1} to X_{2N-1} and the results are compared with the observed (actual) values X_{N+2} to X_{2N}.

$C(p)$ ranges in magnitude between 0 and 1 for uncorrelated and identical distributions respectively.

The evolution of the correlation coefficient with the prediction time has been shown to be a means of determination of the dynamical behavior of the system. For a periodic signal with additive uncorrelated noise, or for uncorrelated noise, the correlation coefficient is time independent. A fall in the correlation with prediction time may indicate either a chaotic signal with or without noise or a random fractal sequence.

Random fractal sequences are a particular class of colored noise with power low spectra $p(\omega) = c(\omega)^{-a}$ which fools other procedure for identifying chaotic behavior : in the Grassberger-Procaccia algorithm they lead to finite correlation dimension [8] though they are true random processes corresponding to an inifinite number of degrees of freedom.

Non linear forecasting allows to go one step further in the distinction between chaos and random fractal sequences[6] : the behavior of $ln(1-C(p))$ is a linear function of p for small p in the case of chaos whereas it is a linear function of $ln(p)$ for random fractal sequences.

We perform prediction for $2N = 500$ points and average the correlation coefficient over the whole enregistrement ($\simeq$ 4000 points). We normalize our data to the range $[-1, +1]$ before performing the prediction. We choose the time delay τ equal to the sampling interval (τ = 10ms) and $\alpha = 0.0005$. Results are presented for $n = 3$.

C. Results and discussion

Eye movements data were recorded with the scleral search coil method providing the horizontal and the vertical components of the eye-position and sampled at 100 Hz [8]. Surgical procedures and microinjection method were similar to those used in the study of Cheron et al. [9]. Fig. 1 and 2 present examples of such results for two types of situations we analyse : the normal awake cat and a pathological eye nystagmic state.

During spontaneous ocular movements like those illustrated in Figure 1, the gaze either jumps rapidly from one point to another (saccade) or remains stable (fixation period). The behavior of the saccadic system shows a unique feature : an invariant relationship (the main sequence) between the peak velocity and the size of the saccade [10]. The bigger the saccade is, the greater its velocity peak is. The frequency of this saccadic movement is dependent on how the central nervous system processes visual information. Young and Stark [11] firstly hypothesized that the saccadic behavior is compatible with a sampled data system but subsequent studies have shown that the sample data model does not adequatly explain some particular saccadic behavior [12]. In fact, the visuo-motor system appears to be able to elaborate program for two saccades at the same time [13]. Whatever the nature of this parallel processing, the main sequence and the frequency of the saccade provide an identifiable scan path by which the eye explores its visual environment.

The non-linear forecasting analysis of a normal scan path (saccades made by an awake cat, Fig. 1) shows a lack of clear dependence of $C(p)$ versus p (Fig. 3). However, $C(p)$ cannot be considered as p independent and we cannot conclude that normal horizontal motion is purely uncorrelated noise. Figure 5 indicates that this scan path does not show either the behavior of random fractal sequences.

Microinjection of an NMDA antagonist into the brainstem oculomotor integrator produces two types of saccadic disorders. The first one is characterized by a post saccadic drift at the end of the saccade: the eye-position cannot be held and the eye drifts toward the primary position. The second one,illustrated in Figure 2, corresponds to a pathologic nystagmus caused by an unsustained eye-position command from the defective neural integrator. Figure 2 illustrates the fact that the eye cannot be held steadily in an eccentric orbital position, but drifts back towards the midline. In this situation, the scan path of the eye is completely distorded, reflecting a clear pathological behavior.

The non-linear forecasting analysis of this pathological scan path firstly reveals in contrast to the normal case a regular decay of the correlation coefficient with prediction time (Fig. 3).

The examination of Fig. 3 and 4 indicates a linear dependence of $ln(1 - C(p))$ versus p for small p. According to the theory [6], this indicates that the time series is chaotic. The slope of $ln(1-C(p))$ against forecasting time is an estimate of K-entropy and measures how chaotic the system is. This scaling behavior has been observed for several time series issued from pathological nystagmic states and for different values of α (cf. Eq. 2). We suggest that the pathological nystagmic pattern results from a reduction of the number of degrees of freedom from the normal case. This encouraging result should be confirmed by other approaches.

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Figure Caption

Fig. 1. Horizontal (solid line) and vertical (dashed line) eye-positions (arbitrary units) versus time for a normal awake cat.

Fig. 2. Horizontal (solid line) and vertical (dashed line) eye- positions (arbitrary units) versus time for a pathological eye nystagmic state.

Fig. 3. Correlation $C(p)$ between forecasted and original time series for horizontal eye-position against prediction time step p. Δ and $\square$ normal awake cats, ▲ and ■ pathologic nystagmus.

Fig. 4. $Ln(1 - C(p))$ against prediction time step p. Δ and $\square$ normal awake cats ▲ and ■ pathologic nystagmus.

Fig. 5. $Ln(1 - C(p))$ agains lnp. Δ and $\square$ normal awake cats, ▲ and ■ pathologic systagmus.

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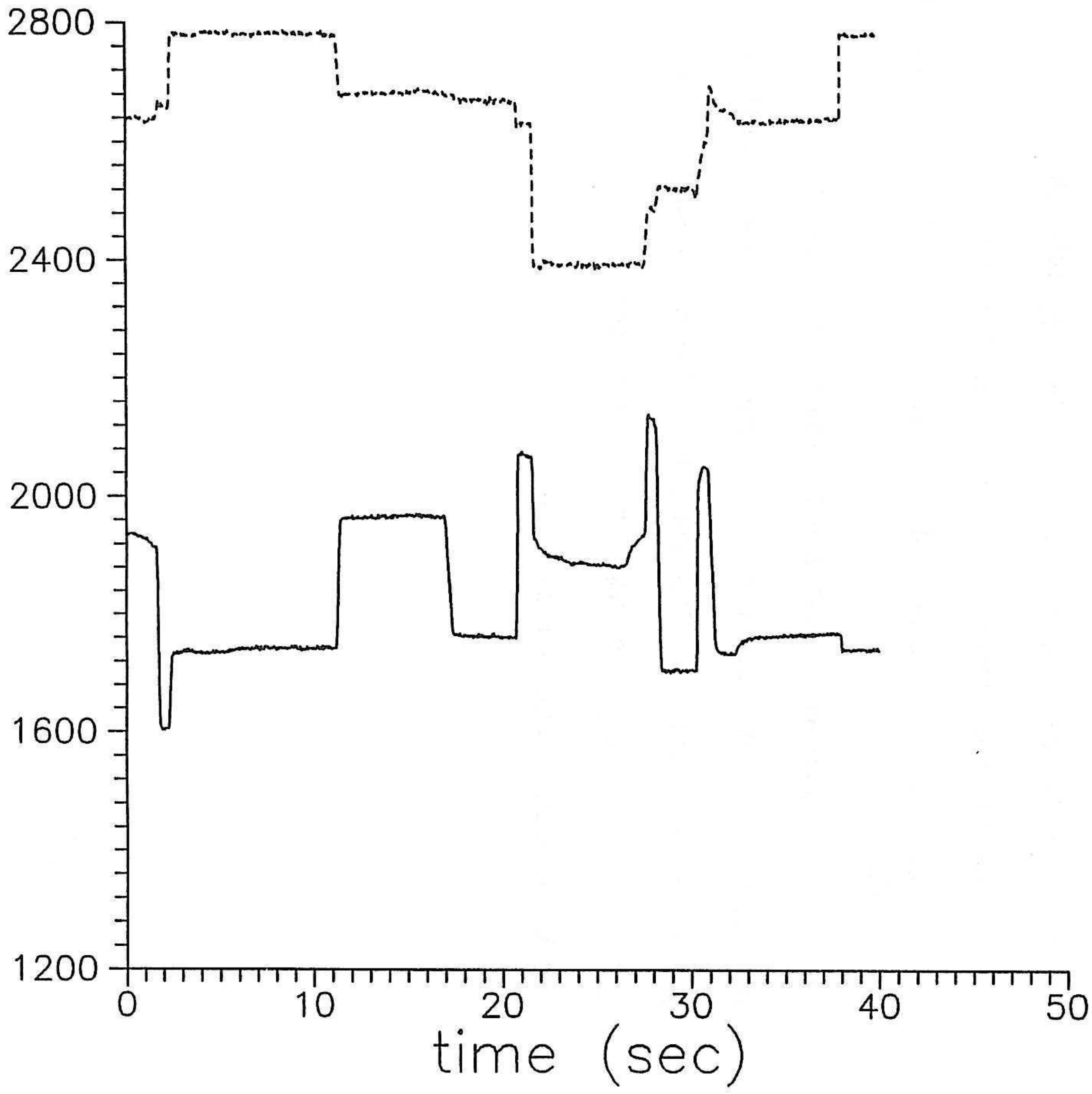

Fig. 1

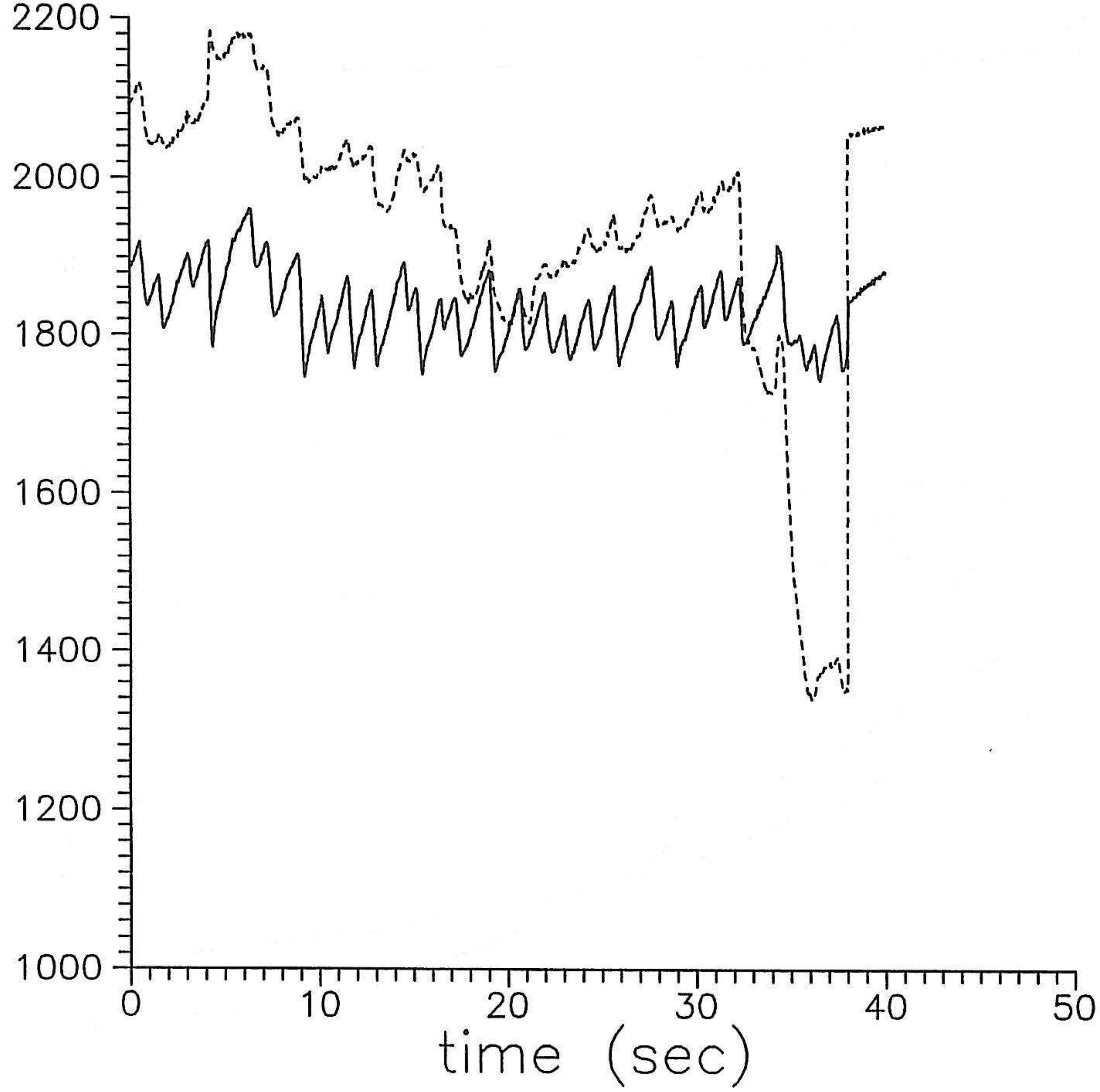

Fig. 2

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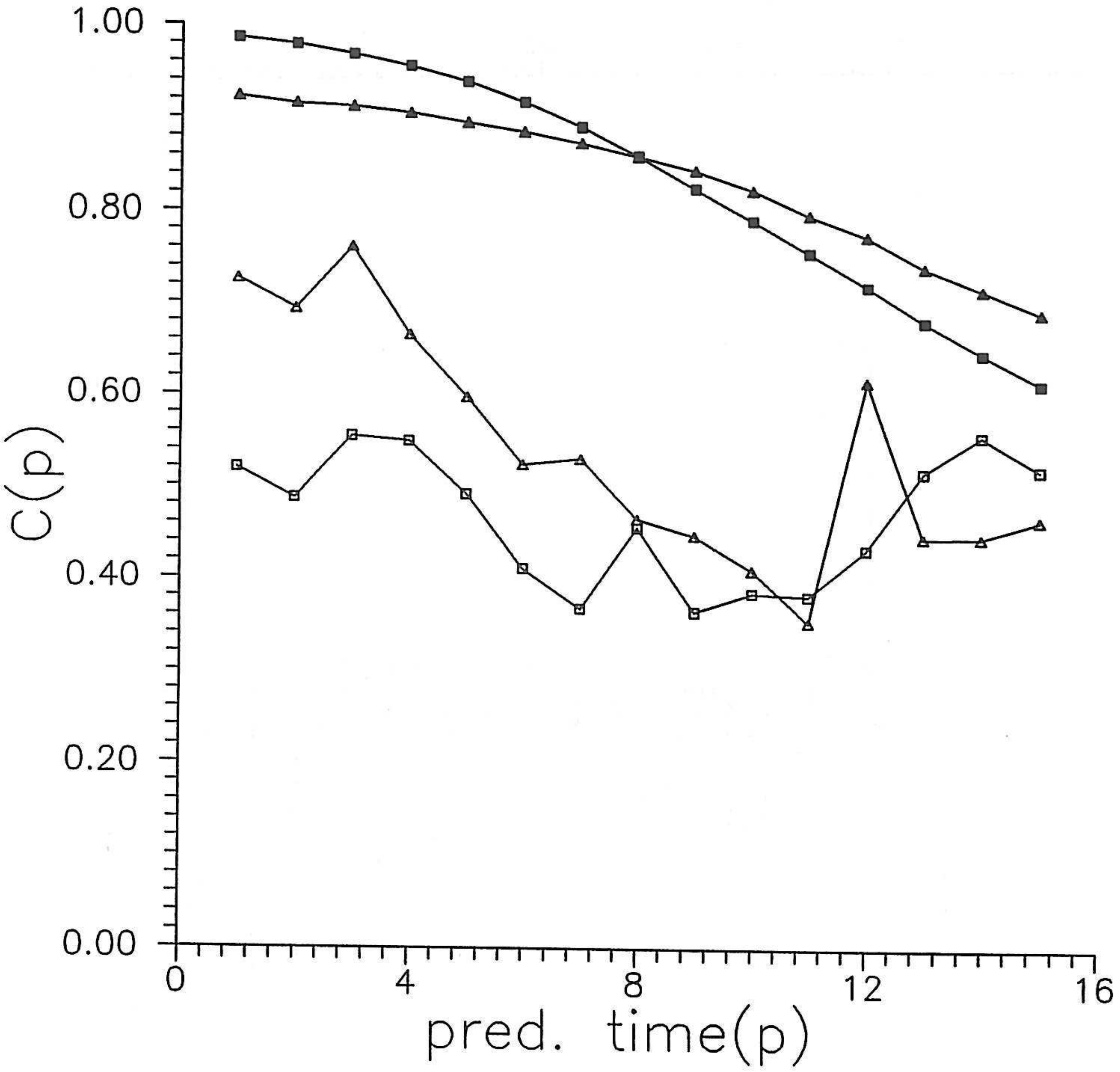

Fig. 3

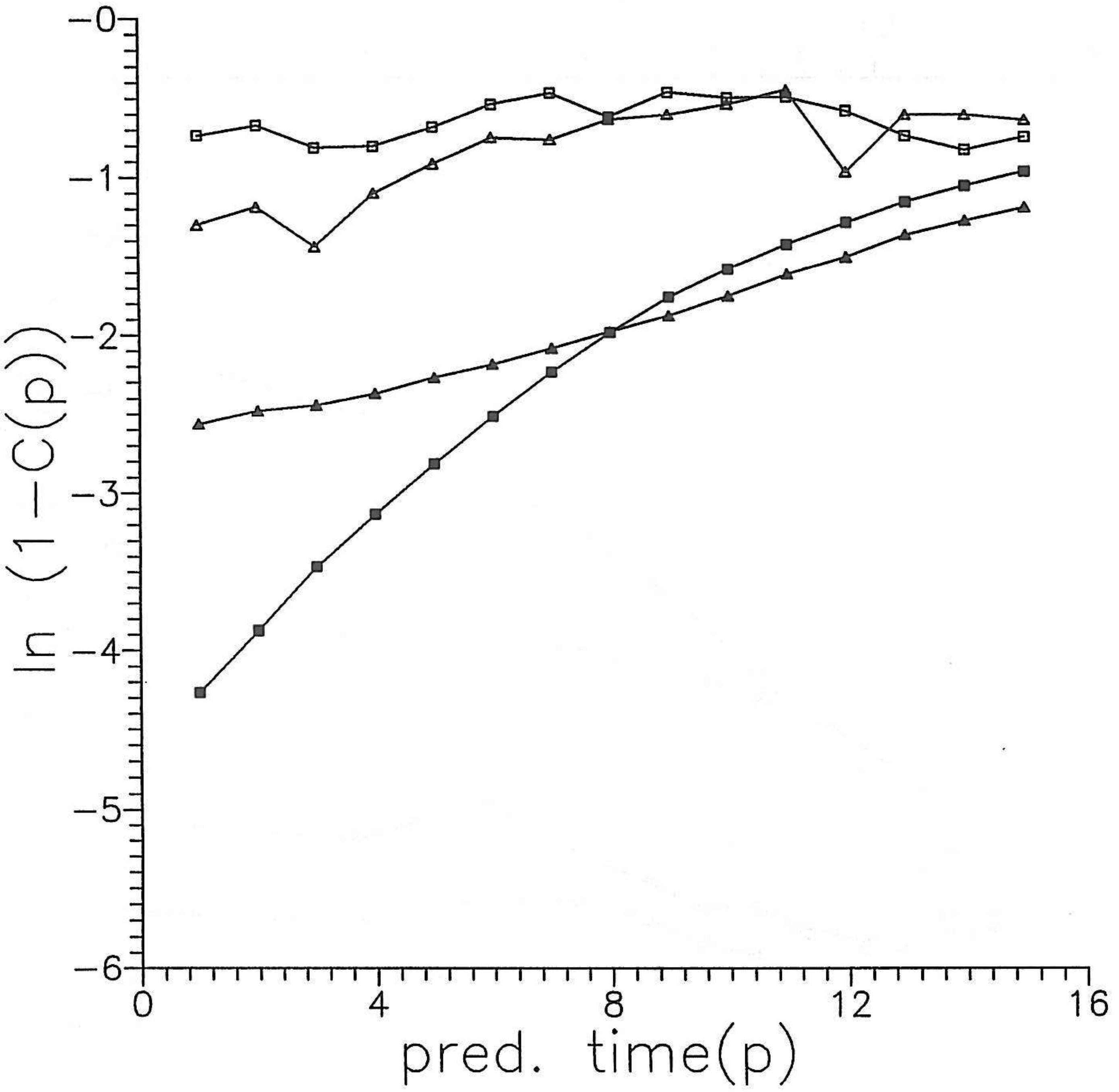

Fig. 4

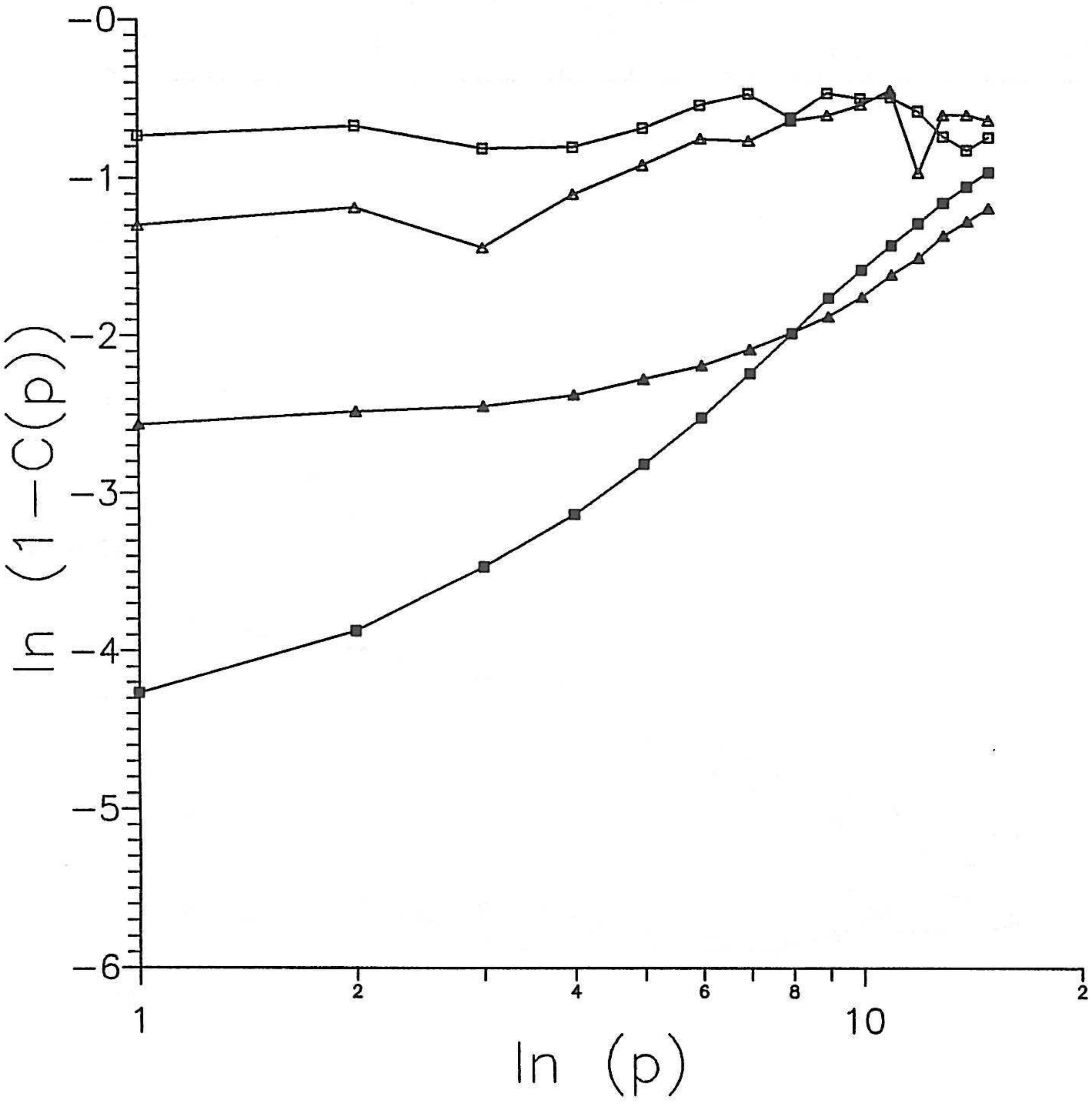

Fig. 5